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Global solutions of real compressible reactive gas with density-dependent viscosity and self-gravitation for higher-order kinetics

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ABSTRACT

A mathematical model for viscous, real, compressible, reactive fluid flows is considered. The existence of global solutions for the free boundary problem with species diffusion in dynamic combustion is established when the viscosity λ depends on the density i.e., $\lambda(\rho) = A\rho^\alpha$ $(0 < \alpha \leqslant \frac{1}{2})$, where A is a generic positive constant. Furthermore, the equations of state depend nonlinearly on density and temperature unlike the case of perfect gases or radiative flows. In addition, the shock wave, turbulence, vacuum, mass concentration or extremely hot spot will not be developed in any finite time if the initial data do not contain vacuum.

1. Introduction

In this paper, we are concerned with a model of real, compressible, reactive gases for the free boundary case with general large initial data in dynamic combustion arising from oil heaters, furnaces, engines, naval fighters or rocket vehicles. This model can be described by the following four equations in the Eulerian coordinate system denoting the conservation laws of the mass, momentum and energy, and an equation of reaction–diffusion type:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + \left(\rho u^2 + p\right)_x = (\lambda u_x)_x + \rho F, \\ \varepsilon_t + \left(u(\varepsilon + p)\right)_x = (\lambda u u_x + k\theta_x + \mu\rho z_x)_x + \rho F u, \\ (\rho z)_t + (\rho u z)_x = (\mu\rho z_x)_x - \rho f(\rho,\theta,z), \quad a(t) < x < b(t),\ t > 0, \end{cases} \tag{1.1}$$

where we denote by ρ the density of the gases, u the velocity, z the mass fraction of the reactant, and θ the absolute temperature. In addition, ε is defined by

$$\varepsilon = \rho\left(e + \frac{1}{2}u^2 + z\right), \tag{1.2}$$

with $e = e(\rho,\theta)$ the internal energy, and f, the intensity of the chemical reaction, is defined by

$$f(\rho,\theta,z) = \delta K\rho^{m-1}z^m \exp\left(\frac{\theta - 1}{\delta\theta}\right) \tag{1.3}$$

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by the Arrhenius law for the positive constants δ, K and $m \geqslant 1$. Moreover, the internal energy e, the pressure p and the heat conduction k are nonlinear functions of density and temperature (see for example (2.11)–(2.13)), which are more general than the perfect flows. On the other hand, the relation between e and p is satisfied by the second thermodynamics law

$$p(\rho, \theta) = \rho^2 \frac{\partial e}{\partial \rho} + \theta \frac{\partial p}{\partial \theta}. \tag{1.4}$$

We emphasize that the viscosity $\lambda(\rho) = A\rho^\alpha$ with positive constants A and α and diffusion term $\mu \geqslant 0$ are completely different from those in the previous works [5–8,11,14,18,23,25]. With no loss of generality, we normalize $A = 1$. For example, we can take the form of $\mu = B\rho^\beta (B, \beta > 0)$ which is different from $\mu = 0$ in [6,8] and $0 < \mu_1 \leqslant \mu(\rho) \leqslant \mu_2$ in [23].

The external force per unit mass $F = F(x,t)$ is determined by $F = -U_x$, where U solves the following boundary value problem

$$\begin{cases} U_{xx} = G\rho, \quad a(t) < x < b(t),\ t > 0, \\ U|_{x=a(t)} = U|_{x=b(t)} = 0, \quad t > 0, \end{cases} \tag{1.5}$$

where G is the Newtonian gravitational constant, and the free boundaries $a(t)$ and $b(t)$ are given by

$$a'(t) = u\big(a(t), t\big), \qquad b'(t) = u\big(b(t), t\big). \tag{1.6}$$

We consider the system (1.1) under the homogeneous free boundary conditions of the form

$$(-p + \lambda u_x)(x,t) = 0, \quad x = a(t), b(t), \tag{1.7}$$

$$(\theta_x, z_x)|(x,t) = 0, \quad x = a(t), b(t), \tag{1.8}$$

and the initial conditions

$$(\rho, u, \theta, z)|_{t=0} = (\rho_0, u_0, \theta_0, z_0)(x). \tag{1.9}$$

We are interested in the global existence of real, compressible, reactive fluid flows with homogeneous free boundary conditions when the viscosity depends on density under gravitation. From the physical point of view, the viscosity of the realistic model may depend on density, temperature, or the mass fraction of the reaction. Restricted by the mathematical techniques, however, many scholars only studied various simplified model to simulate the real word. For example, Kawohl [17] and Wang [21,22] showed the global existence of solutions for non-reactive real compressible gas in which the viscosity was a function of density, which was bounded from below and above. Then Wang [23] extended their results to the case of reactive viscous compressible flows. Jiang [15] proved the global existence of the free boundary problem in the case of $\lambda(\rho) = A\rho^\alpha$ $(0 < \alpha < \frac{1}{4})$ for non-reactive gas, which recently is extended to $0 < \alpha \leqslant \frac{1}{2}$ for the γ-type pressure law by Qin and Yao [20]. The common point of these papers is that the influence of the external force is not considered. In a very recent paper [24], M. Umehara and A. Tani studied the global existence of solutions to a self-gravitating viscous radiative and reactive gas with nonzero outer pressure on the free boundaries in the case of first-order kinetics only $(m = 1)$ in one-dimensional space when the viscosity is constant. We remark that the nonzero outer pressure provides some damping effect and keeps the volume of the gas bounded all the time. Therefore, it is not clear that the volume of the gas will keep bounded for ever in the case of homogeneous free boundary conditions. Particularly, the present paper covers the case of higher-order kinetics $(m > 1)$.

For the study of the reactive gas, the earlier works focused on the case of constant viscosity and perfect flows, see [2–4] by A. Bressan and J. Bebernes for the global existence, [25] by Yanagi and [14] by B. Guo and P. Zhu for the stability and asymptotic behaviors of solutions and other related works for example [5–7] and the references cited therein.

Taking into account the process of combustion, the temperature of reactant changes rapidly from the induction stage to explosion stage and finally approaches chemical equilibrium. Thus, the equations of state of perfect gases are inadequate for high temperature regime, see [1,26] for more detailed explanation and some experiment facts for more physical background. Based on this consideration, Wang [23] first showed the existence and large time behavior of global solutions for the real compressible reactive gases under the Dirichlet–Neumann boundary conditions when the viscosity is assumed to be bounded from below and above for the real gas. Moreover, motivated by the ideas of [18], M. Lewicka and P.B. Mucha [19] investigated the long time behavior of solutions for reactive gas confined between two parallel plates with the constant viscosity and the pressure term given by

$$p(\rho, \theta) = \rho^\gamma \theta, \quad \gamma \geqslant 1. \tag{1.10}$$

It is clear that (1.10) is more general than the perfect gas. We take advantage of (1.10) and make more general form of pressure, which can include the case of [23], refer to (2.12).

Motivated by the previous works, we consider the existence of global solutions when the viscosity $\lambda(\rho) = A\rho^\alpha$ with $0 < \alpha \leqslant \frac{1}{2}$ and the diffusion term $\mu \geqslant 0$ with more general pressure law than (1.10). As an example, one of the best known approximations in a real gas is the *Beattie–Bridgman* state equation given by

$$p(\rho,\theta) = R\rho\theta + \beta_1\rho^2 + \beta_2\rho^3 + \beta_3\rho^4, \tag{1.11}$$

where β_i are constants (refer to [1,9,12]), which is a special case what we consider. Furthermore, under our considerations, the viscosity will decrease rapidly as the density tends to zero. This fact leads to new difficulty to yield various estimates on the solution (ρ, u, θ, z), especially the bounds of density. On the other hand, the influence of the self-gravitation for the real reactive gas is also considered. Thus, the ideas of [15,16,23] cannot be adopted. Therefore, we develop some new techniques and methods in Lagrangian coordinate. To complete the proof for main result, we first obtain an entropy energy estimate involving the dissipative effects of viscosity and heat diffusion. Then we show the estimate of specific volume on the free boundaries and some auxiliary lemmas to get the upper and lower bound of density, especially, the lower bound. Finally, the existence and uniqueness of local solutions can be obtained by the standard method, see [17] for details. With the global estimate at hand, the global existence of solutions is proved by extending the local solutions globally in time.

Last not at least, we refer to some new progresses on the multidimensional reacting, radiative model with more various assumptions on pressure, viscosity, see [9,10,13] for the Dirichlet problem and the references cited therein.

The outline of the paper is as follows. In Section 2, we introduce the Lagrangian coordinate and transform the problem (1.1), (1.7)–(1.9) into the fixed boundary problem which is equivalent under consideration. The main result is also stated in this part. In Section 3, we make some a priori estimates to prove the main result by extending the local solutions globally in time.

2. Reformation of the free boundary problem

To solve the free boundary problem (1.1), (1.7)–(1.9) more conveniently, we introduce the Lagrangian variable, i.e.,

$$y = \int_{a(t)}^{x} \rho(\xi,t)\,d\xi, \quad t = t. \tag{2.1}$$

The corresponding form of (1.5) is

$$\begin{cases} (\rho U_y)_y = G, & (y,t) \in (0,1)\times(0,\infty), \\ U|_{y=0} = U|_{y=1} = 0, & t>0, \end{cases} \tag{2.2}$$

and F can be computed by the relation $F = -\rho U_y$, i.e.,

$$F(y,t) = -G\left(y - \frac{\int_0^1 y\rho^{-1}(y,t)\,dy}{\int_0^1 \rho^{-1}(y,t)\,dy}\right). \tag{2.3}$$

The system (1.1) becomes

$$\rho_t + \rho^2 u_y = 0, \tag{2.4a}$$

$$u_t + p_y = (\lambda\rho u_y)_y + F, \tag{2.4b}$$

$$E_t + (up)_y = \big(\rho(\lambda u u_y + k\theta_y + \mu\rho z_y)\big)_y + Fu, \tag{2.4c}$$

$$z_t + f(\rho,\theta,z) = \big(\mu\rho^2 z_y\big)_y, \quad 0<y<1,\ t>0, \tag{2.4d}$$

where $E = e + \frac{u^2}{2} + z$.

We get by integration from (2.2) and (2.4b)

$$\frac{d}{dt}\int_0^1 u\,dy = -G\left(\frac{1}{2} - \frac{\int_0^1 y\rho^{-1}(y,t)\,dy}{\int_0^1 \rho^{-1}(y,t)\,dy}\right). \tag{2.5}$$

If we denote $v = 1/\rho$ and $u - \int_0^1 u\,dy$ by u again, we have

$$v_t - u_y = 0, \tag{2.6a}$$

$$u_t + p_y = \left(\frac{\lambda u_y}{v}\right)_y - G\left(y - \frac{1}{2}\right), \tag{2.6b}$$

$$E_t + (up)_y = \left(\frac{\lambda u u_y + k\theta_y}{v} + \frac{\mu z_y}{v^2}\right)_y - G\left(y - \frac{1}{2}\right)u, \tag{2.6c}$$

$$z_t + f(\rho,\theta,z) = \left(\frac{\mu z_y}{v^2}\right)_y, \quad 0<y<1,\ t>0. \tag{2.6d}$$

The corresponding boundary conditions are

$$(\theta_y, z_y)|_{y=0,1} = 0 \tag{2.7}$$

and

$$\big(\lambda(\rho)\rho u_y - p\big)\big|_{y=0,1} = 0, \tag{2.8}$$

and the initial data is

$$(\rho, u, \theta, z)|_{t=0} = (\rho_0, u_0, \theta_0, z_0)(y). \tag{2.9}$$

Obviously, u satisfies

$$\int_0^1 u\,dy = \int_0^1 u_0(y)\,dy = 0. \tag{2.10}$$

To comply with the physical models (see [1,26]), we assume that $p \geqslant 0$ and $e \geqslant 0$ are continuously differentiable and $k \geqslant 0$ is twice continuously differential. We also suppose some physical growth conditions on p, e and k: There are constants ν, P_1, P_2, k_0, such that for any given $C > 0$, there exist positive constants $N(C)$, $k(C)$ and $k_1(C)$, such that for $v > 0$, $\theta \geqslant 0$, the following conditions hold:

$$e(v,0) \geqslant 0, \quad \nu\big(1+\theta^r\big) \leqslant e_\theta(v,\theta) \leqslant N(C)\big(1+\theta^r\big); \tag{2.11}$$

$$p(v,\theta) \geqslant 0, \qquad p(v,\theta) \to 0, \quad \text{as } v \to \infty, \tag{2.12a}$$

$$\big|p_\theta(v,\theta)\big| \leqslant N(C)\frac{1+\theta^r}{v^\gamma}, \tag{2.12b}$$

$$-\frac{P_2(\beta + (1-\beta)\theta + \theta^{1+r})}{v^{\gamma+1}} \leqslant p_v(v,\theta) \leqslant -\frac{P_1(\beta + (1-\beta)\theta + \theta^{1+r})}{v^{\gamma+1}}; \tag{2.12c}$$

$$k_0\big(1+\theta^q\big) \leqslant k(v,\theta) \leqslant k_1(C)\big(1+\theta^q\big), \tag{2.13a}$$

$$\big|k_v(v,\theta)\big| + \big|k_{vv}(v,\theta)\big| \leqslant k_1(C)\big(1+\theta^q\big), \tag{2.13b}$$

where $r \in [0,1]$ and $q \geqslant 2+2r$, $\gamma \geqslant 1$, $\beta = 0$ or 1.

Remark 2.1. By integrating (2.12c) over (v,∞) and noticing (2.12a), we obtain

$$\frac{P_1[\beta + (1-\beta)\theta + \theta^{1+r}]}{\gamma} \leqslant v^\gamma p(v,\theta) \leqslant \frac{P_2[\beta + (1-\beta)\theta + \theta^{1+r}]}{\gamma}. \tag{2.14}$$

Theorem 2.1. *Let $0 < \alpha \leqslant \frac{1}{2}$, and* (2.11)–(2.13) *hold. Assume that $v_0 \in W^{1,\infty}(0,1)$, and there exists a constant $C_0 > 0$ such that*

$$C_0^{-1} \leqslant v_0, \qquad \theta_0 \leqslant C_0, \qquad \big\|(v_0, u_0, \theta_0, z_0)\big\|_{H^1} \leqslant C_0, \quad 0 \leqslant z_0 \leqslant 1. \tag{2.15}$$

Then there exists a unique global solution (v, u, θ, z) to the initial boundary value problem (2.6)–(2.9)*, such that for any fixed time $T > 0$ and each $(y,t) \in [0,1] \times [0,T]$,*

$$C^{-1} \leqslant v(y,t), \qquad \theta(y,t) \leqslant C, \qquad 0 \leqslant z(y,t) \leqslant 1, \tag{2.16}$$

$$v \in L^\infty\big(0,T; H^1 \cap W^{1,\infty}([0,1])\big), \qquad (u,\theta,z) \in L^\infty\big(0,T; H^1([0,1])\big), \tag{2.17}$$

and

$$\big\|(v,u,\theta,z)\big\|_{H^1}^2(t) + \int_0^t \big\|(v_{yt}, u_{yy}, \theta_{yy}, z_{yy})\big\|_{L^2}^2(s)\,ds \leqslant C, \tag{2.18}$$

where C denotes positive constants.

Remark 2.2. The same ideas apply to the case of one boundary is fixed and the other is free.

Remark 2.3. We emphasize that the technique can adopt to [20] to yield the corresponding result when the gravitation is considered.

Remark 2.4. Indeed, with the bounds of the density, we can describe the behavior of the free boundaries as the time tends to infinity by the same method as in [22].

Remark 2.5. We can generalize the result to the corresponding initial boundary problem with the initial in a Hölder space or with discontinuous initial data by mollification in the case of perfect flows [21].

3. Proof of Theorem 2.1

In this section, we will give the proof of Theorem 2.1, which is based upon the continuation of a local solution. First, we obtain some a priori estimates on the state parameters (ρ, u, θ, z), especially the upper and the lower estimates for ρ. These will allow us to extend the local solution globally in time.

In the sequel, we will denote $C(C(T))$ generic positive constant, which may depend on the initial data and other indexes in the governing equations (fixed time T).

Lemma 3.1. *Under the assumptions of Theorem* 2.1*, we have*

$$0 \leqslant z(y,t) \leqslant \max_{y\in[0,1]} z_0(y) \leqslant 1, \quad \forall (y,t) \in [0,1]\times[0,T]. \tag{3.1}$$

Proof. The proof follows from (2.6d) and the maximum principle. For detailed proof, we refer to [23]. □

Lemma 3.2. *One has*

$$\int_0^1 \left(\theta + \theta^{1+r} + \frac{u^2}{2} + z\right) dy \leqslant C, \quad \forall 0 < t \leqslant T. \tag{3.2}$$

Proof. Integrating (2.6c) and noticing the boundary conditions (2.7) and (2.8), we get

$$\int_0^1 E\,dy = \int_0^1 E_0\,dy - G\int_0^t\int_0^1 \left(y - \frac{1}{2}\right) u\,dy\,ds \leqslant C(T) + C\int_0^t\int_0^1 u^2\,dy\,ds,$$

by Young's inequality. From (2.11), we arrive at

$$\int_0^1 \left(\theta + \theta^{1+r} + \frac{u^2}{2} + z\right) dy \leqslant C(T) + C\int_0^t\int_0^1 u^2\,dy\,ds,$$

which implies

$$\int_0^1 u^2\,dy \leqslant C(T),$$

by Gronwall inequality.

This completes the proof of the lemma. □

The following lemma gives the upper bound of the density.

Lemma 3.3. *We have*

$$\rho(y,t) \leqslant C(T), \quad \forall (y,t) \in [0,1]\times[0,T]. \tag{3.3}$$

Proof. It follows from (2.6a) and (2.6b) that

$$\rho^\alpha_{ty} = -\alpha(u_t + p_y).$$

Integrating it over $[0,t]\times[0,y]$ yields

$$\begin{aligned}\rho^\alpha(y,t)+\rho^\alpha(0,0)&=\rho_0^\alpha(y)-\alpha\int_0^y\big(u-u_0(x)\big)\,dx-\alpha\int_0^t p(y,s)\,ds-G\int_0^t\int_0^1\Big(x-\frac{1}{2}\Big)\,dx\,ds\\&\leqslant C(T)+C\int_0^1 u^2\,dy+C\int_0^1 u_0^2\,dy\leqslant C(T),\end{aligned}$$

with the aid of Young inequality and Lemma 3.2.

This completes the proof of the lemma. □

The following lemma plays an important role in the context since it involves the dissipative effects of viscosity and heat diffusion.

Lemma 3.4. *Under the assumptions of Theorem* 2.1, *there holds*

$$\int_0^t\int_0^1\frac{\rho(1+\theta^q)\theta_y^2}{\theta^2}\,dy\,ds+\int_0^t\int_0^1\frac{\rho^{1+\alpha}u_y^2}{\theta}\,dy\,ds+\int_0^t\int_0^1\frac{f}{\theta}\,dy\,ds\leqslant C. \tag{3.4}$$

Proof. Define $\psi(v,\theta)=e(v,\theta)-\theta\eta(v,\theta)$, where $e_\theta=\theta\eta_\theta$, $\eta_v=p_\theta$. Some direct calculations yield

$$\psi_v(v,\theta)=-p,\qquad e_\theta=-\theta\psi_{\theta\theta}. \tag{3.5}$$

From (1.3) and (2.6), we obtain

$$\begin{aligned}&\Big[\psi(v,\theta)-\psi(v,1)-\psi_\theta(v,\theta)(\theta-1)+\frac{u^2}{2}+z\Big]_t+\frac{k\rho\theta_y^2}{\theta^2}+\frac{\rho^{\alpha+1}u_y^2}{\theta}+\frac{f}{\theta}\\&\quad=p(v,1)u_y+\Big[\rho\Big(\rho^\alpha uu_y+k\theta_y+\mu\rho z_y-\frac{k\theta_y}{\theta}\Big)-vp\Big]_y,\end{aligned}$$

from which it follows, by integration, that

$$\begin{aligned}&\int_0^1\big[\psi(v,\theta)-\psi(v,1)-\psi_\theta(v,\theta)(\theta-1)\big]\,dy+\int_0^1\Big(\frac{u^2}{2}+z\Big)\,dy+\int_0^t\int_0^1\Big(\frac{k\rho\theta_y^2}{\theta^2}+\frac{\rho^{\alpha+1}u_y^2}{\theta}+\frac{f}{\theta}\Big)\,dy\,ds\\&\quad\leqslant C+\frac{2P_2}{\gamma}\int_0^t\int_0^1\frac{|u_y|}{v^\gamma}\,dy\,ds.\end{aligned}$$

By Young's inequality, it yields

$$\begin{aligned}&\int_0^1\big[\psi(v,\theta)-\psi(v,1)-\psi_\theta(v,\theta)(\theta-1)\big]\,dy+\int_0^1\Big(\frac{u^2}{2}+z\Big)\,dy+\int_0^t\int_0^1\Big(\frac{k\rho\theta_y^2}{\theta^2}+\frac{\rho^{\alpha+1}u_y^2}{\theta}+\frac{f}{\theta}\Big)\,dy\,ds\\&\quad\leqslant C+C\int_0^t\int_0^1\theta v^{\alpha+1-2\gamma}\,dy\,ds\leqslant C,\end{aligned}\tag{3.6}$$

where we used Lemmas 3.2–3.3. From Taylor's expansion theorem, one gets

$$\psi(v,\theta)-\psi(v,1)-\psi_\theta(v,\theta)(\theta-1)\geqslant\nu(\theta-1)^2\int_0^1\frac{(1-s)[1+(\theta+s(1-\theta))^r]}{\theta+s(1-\theta)}\,ds\geqslant 0,$$

which implies the desired result by Lemma 3.1. □

Lemma 3.5. *Under the hypotheses of Theorem* 2.1, *there holds*

$$\int_0^1 v^{1-\alpha}\,dy\leqslant C(T),\quad\forall t\in(0,T]. \tag{3.7}$$

Proof. Multiplying (2.6b) by v and integrating over $[0, y]$ yield

$$\frac{d}{dt}\int_0^v \lambda(\xi)\,d\xi = vp + v\int_0^y u_t\,dx - Gv\int_0^y \left(x-\frac{1}{2}\right)dx.$$

Integrating the above equality over $[0,1]\times[0,t]$ yields

$$\frac{1}{1-\alpha}\left(\int_0^1 v^{1-\alpha}\,dy - \int_0^1 v_0^{1-\alpha}\,dy\right) = \int_0^t\int_0^1 vp\,dy\,ds + \int_0^t\int_0^1 v\left(\int_0^y u_t\,dx\right)dy\,ds + \frac{G}{2}\int_0^t\int_0^1 v\left[\left(y-\frac{1}{2}\right)^2-\frac{1}{4}\right]dy\,ds. \tag{3.8}$$

By (2.14) and Lemmas 3.2–3.3, we have

$$\int_0^t\int_0^1 vp\,dy\,ds \leqslant \frac{P_2}{\gamma}\int_0^t\int_0^1 \frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^{\gamma-1}}\,dy\,ds \leqslant C(T). \tag{3.9}$$

Recalling (2.10), a calculation leads to

$$\begin{aligned}
\int_0^t\int_0^1 v\left(\int_0^y u_t\,dx\right)dy\,ds &= \int_0^t\int_0^1 \frac{d}{ds}\left(v\int_0^y u\,dx\right)dy\,ds - \int_0^t\int_0^1 v_s\left(\int_0^y u\,dx\right)dy\,ds\\
&= \int_0^1 v\left(\int_0^y u\,dx\right)dy - \int_0^1 v_0(y)\left(\int_0^y u_0(x)\,dx\right)dy - \int_0^t\int_0^1 u_y\left(\int_0^y u\,dx\right)dy\,ds\\
&= \int_0^1 v\left(\int_0^y u\,dx\right)dy - \int_0^1 v_0(y)\left(\int_0^y u_0(x)\,dx\right)dy + \int_0^t\int_0^1 u^2\,dy\,ds,
\end{aligned}$$

and by $v(y,t) = v_0(y) + \int_0^t u_y(y,s)\,ds$, we deduce

$$\begin{aligned}
\int_0^t\int_0^1 v\left(\int_0^y u_t\,dx\right)dy\,ds &= \int_0^1 v_0(y)\left(\int_0^y \big(u-u_0(x)\big)\,dx\right)dy + \int_0^1\left(\int_0^t u\,ds\right)_y\left(\int_0^y u\,dx\right)dy + \int_0^t\int_0^1 u^2\,dy\,ds\\
&= \int_0^1 v_0(y)\left(\int_0^y \big(u-u_0(x)\big)\,dx\right)dy - \int_0^1 u(y,t)\left(\int_0^t u(y,s)\,ds\right)dy + \int_0^t\int_0^1 u^2\,dy\,ds.
\end{aligned} \tag{3.10}$$

Noticing $(y-\frac{1}{2})^2 - \frac{1}{4} \leqslant 0$ on $[0,1]$ and $v(y,t) > 0$ on $[0,1]\times[0,T]$, we get

$$\frac{G}{2}\int_0^t\int_0^1 v\left[\left(y-\frac{1}{2}\right)^2-\frac{1}{4}\right]dy\,ds \leqslant 0. \tag{3.11}$$

Collecting (3.8)–(3.11), we arrive at

$$\begin{aligned}
\int_0^1 v^{1-\alpha}\,dy &\leqslant C(T) + C\int_0^1 v_0^{1-\alpha}(y)\,dy + \int_0^1 v_0(y)\left(\int_0^y \big(u-u_0(x)\big)\,dx\right)dy - \int_0^1 u(y,t)\left(\int_0^t u(y,s)\,ds\right)dy + \int_0^t\int_0^1 u^2\,dy\,ds\\
&\leqslant C(T) + C\int_0^1 v_0^{1-\alpha}(y)\,dy + C\int_0^1 v_0^2(y)\,dy + C\int_0^1 u_0^2(y)\,dy + C\int_0^1 u^2\,dy + C\int_0^t\int_0^1 u^2\,dy\,ds\\
&\leqslant C(T),
\end{aligned}$$

by the initial conditions and Lemma 3.2.

This completes the proof of the lemma. □

The following lemma gives the estimate of density on the free boundaries.

Lemma 3.6. *We have*

$$v^{\alpha}(d,t) \leqslant C(T) + C(T)\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds \tag{3.12}$$

for $d = 0, 1$.

Proof. It follows from the boundary conditions (2.6a) and (2.8) that

$$\left(\frac{1}{1-\alpha}\left(\rho^{\alpha-1}\right)_t\right)(d,t) = (vp)(d,t), \quad d = 0, 1. \tag{3.13}$$

Integrating (3.13) over $[0,t]$ and recalling (2.14), we have

$$v^{1-\alpha}(d,t) \leqslant \frac{P_2(1-\alpha)}{\gamma}\int_0^t \frac{(\beta+(1-\beta)\theta+\theta^{1+r})(d,s)}{v^{\gamma-1}}\,ds + v_0^{1-\alpha}(d).$$

Since $0 < \alpha \leqslant \frac{1}{2}$, we obtain

$$v^{\alpha}(d,t) \leqslant \left[\frac{P_2(1-\alpha)}{\gamma}\int_0^t \frac{(\beta+(1-\beta)\theta+\theta^{1+r})(d,s)}{v^{\gamma-1}}\,ds + v_0^{1-\alpha}(d)\right]^{\frac{\alpha}{1-\alpha}} \leqslant C(T) + C(T)\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds, \tag{3.14}$$

by Young's inequality and Lemma 3.3. This finishes the proof. □

Lemma 3.7. *There holds*

$$\int_0^1 \left[\left(\rho^{\alpha}\right)_y\right]^2 dy \leqslant C(T) + C(T)\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds, \tag{3.15}$$

where $0 < \alpha \leqslant \frac{1}{2}$.

Proof. Set

$$M(v) = \int_{\inf_{[0,1]} v_0}^{v} \frac{\lambda(\xi)}{\xi}\,d\xi. \tag{3.16}$$

Multiplying (2.6b) by $M(v)_y - u$ and integrating it over $[0,1]\times[0,t]$, one has

$$\int_0^1 \left(M(v)_y - u\right)^2(y,t)\,dy \leqslant C + \int_0^t\int_0^1 \left(p_v v_y + p_\theta \theta_y\right)\left(M(v)_y - u\right)dy\,ds - G\int_0^t\int_0^1 \left(y - \frac{1}{2}\right)\left(M(v)_y - u\right)dy\,ds. \tag{3.17}$$

To finish up the proof, we need to give the estimates of the integral on the right-hand side of (3.17). By (3.16), we find

$$\int_0^t\int_0^1 p_v v_y\left(M(v)_y - u\right)dy\,ds = \int_0^t\int_0^1 p_v \frac{v}{\lambda(v)} M(v)_y\left\{M(v)_y - u\right\}dy\,ds.$$

Using (2.12c) and Young's inequality, one has

$$\begin{aligned}
&\int_0^t\int_0^1 p_v v_y\left(M(v)_y - u\right)dy\,ds \\
&\quad\leqslant -P_1\int_0^t\int_0^1 \frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^{\gamma}\lambda(v)} M(v)_y^2\,dy\,ds + P_2\int_0^t\int_0^1 \frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^{\gamma}\lambda(v)}\left|M(v)_y\right||u|\,dy\,ds \\
&\quad\leqslant -\frac{P_1}{2}\int_0^t\int_0^1 \frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^{\gamma}\lambda(v)} M(v)_y^2\,dy\,ds + C\int_0^t\int_0^1 \frac{(1+\theta^{1+r})}{v^{\gamma}\lambda(v)} u^2\,dy\,ds.
\end{aligned}$$

From Lemmas 3.2–3.3 and $0<\alpha\leqslant 1/2$, $\gamma\geqslant 1$, we get

$$\begin{aligned}\int_0^t\int_0^1 p_v v_y\big(M(v)_y-u\big)\,dy\,ds &\leqslant -\frac{P_1}{2}\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds\\ &\quad+C\int_0^t\int_0^1\frac{u^2}{v^{\gamma-\alpha}}\,dy\,ds+C\int_0^t\max_{y\in[0,1]}\theta^{1+r}(y,s)\left(\int_0^1\frac{u^2}{v^{\gamma-\alpha}}\,dy\right)ds\\ &\leqslant -\frac{P_1}{2}\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds+C(T)+C\int_0^t\max_{y\in[0,1]}\theta^{1+r}(y,s)\,ds.\end{aligned}\tag{3.18}$$

Following the same procedure and recalling (2.12b), we arrive at

$$\begin{aligned}&\int_0^t\int_0^1 p_\theta\theta_y\big(M(v)_y-u\big)\,dy\,ds\\ &\quad\leqslant N(C)\int_0^t\int_0^1\frac{(1+\theta^r)|\theta_y|}{v^\gamma}\big(\big|M(v)_y\big|+|u|\big)\,dy\,ds\\ &\quad\leqslant\frac{P_1}{4}\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds+C\int_0^t\int_0^1\frac{\lambda(v)(1+\theta^r)^2}{v^\gamma(\beta+(1-\beta)\theta+\theta^{1+r})}\theta_y^2\,dy\,ds\\ &\qquad+C\int_0^t\int_0^1\frac{(1+\theta^{1+r})}{v^\gamma\lambda(v)}u^2\,dy\,ds\\ &\quad\leqslant\frac{P_1}{4}\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds+C\int_0^t\int_0^1\frac{\theta(1+\theta^r)\theta_y^2}{v\theta^2}\,dy\,ds+C\int_0^t\int_0^1\frac{u^2}{v^{\gamma-\alpha}}\,dy\,ds\\ &\qquad+C\int_0^t\max_{y\in[0,1]}\theta^{1+r}(y,s)\left(\int_0^1\frac{u^2}{v^{1-\alpha}}\,dy\right)ds,\end{aligned}\tag{3.19}$$

where we used the following inequality

$$\frac{(1+\theta^r)^2}{(\beta+(1-\beta)\theta+\theta^{1+r})}\leqslant C\frac{1+\theta^r}{\theta}\quad(\beta=0,1).$$

Therefore, one gets

$$\begin{aligned}\int_0^t\int_0^1 p_\theta\theta_y\big(M(v)_y-u\big)\,dy\,ds &\leqslant\frac{P_1}{4}\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds+C\int_0^t\int_0^1\frac{(1+\theta^q)\theta_y^2}{v\theta^2}\,dy\,ds+C(T)\\ &\quad+C\int_0^t\max_{y\in[0,1]}\theta^{1+r}(y,s)\,ds\quad\big(q\geqslant 2(1+r)\big)\\ &\leqslant\frac{P_1}{4}\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds+C(T)+C\int_0^t\max_{y\in[0,1]}\theta^{1+r}(y,s)\,ds.\end{aligned}\tag{3.20}$$

Combining (3.17)–(3.20), we have

$$\begin{aligned}&\int_0^1\big(M(v)_y-u\big)^2(y,t)\,dy+\int_0^t\int_0^1\frac{(\beta+(1-\beta)\theta+\theta^{1+r})}{v^\gamma\lambda(v)}M(v)_y^2\,dy\,ds\\ &\quad\leqslant C(T)+C\int_0^t\max_{y\in[0,1]}\theta^{1+r}(y,s)\,ds-G\int_0^t\int_0^1\left(y-\frac{1}{2}\right)\big(M(v)_y-u\big)\,dy\,ds\end{aligned}$$

$$\leqslant C(T) + C\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds + C\int_0^t\int_0^1 \big(M(v)_y - u\big)^2(y,t)\,dy\,ds,$$

which and Gronwall's inequality imply that

$$\int_0^1 \big(M(v)_y - u\big)^2(y,t)\,dy \leqslant C(T) + C\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds, \tag{3.21}$$

i.e.,

$$\int_0^1 \big[(\rho^\alpha)_y\big]^2\,dy \leqslant C(T) + C\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds,$$

which completes the proof. □

Lemma 3.8. *We have*

$$\int_0^t \|\rho^\alpha u\|_{C[0,1]}\,ds \leqslant C(T) + C(T)\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds, \quad \forall t\in(0,T].$$

Proof. From the embedding theorem $W^{1,1}[0,1] \hookrightarrow C[0,1]$, we obtain

$$\begin{aligned}\int_0^t \|\rho^\alpha u\|_{C[0,1]}\,ds &\leqslant C\int_0^t\int_0^1 |\rho^\alpha u|\,dy\,ds + C\int_0^t\int_0^1 \big|(\rho^\alpha u)_y\big|\,dy\,ds\\ &\leqslant C\int_0^t\int_0^1 |\rho^\alpha u|\,dy\,ds + C\int_0^t\int_0^1 \big|(\rho^\alpha)_y u\big|\,dy\,ds + C\int_0^t\int_0^1 \rho^\alpha |u_y|\,dy\,ds.\end{aligned}$$

Thus, by Young's inequality, we have

$$\begin{aligned}\int_0^t \|\rho^\alpha u\|_{C[0,1]}\,ds &\leqslant C\int_0^t\int_0^1 \rho^{2\alpha}\,dy\,ds + C\int_0^t\int_0^1 (\rho^\alpha)_y^2\,dy\,ds + C\int_0^t\int_0^1 u^2\,dy\,ds\\ &\quad + C\int_0^t\int_0^1 \frac{\rho^{1+\alpha}u_y^2}{\theta}\,dy\,ds + C\int_0^t\int_0^1 \rho^{\alpha-1}\theta\,dy\,ds\\ &\leqslant C(T) + C\int_0^t\left(\int_0^s \max_{y\in[0,1]} \theta^{1+r}(y,\tau)\,d\tau\right)ds + \int_0^t \max_{y\in[0,1]} \theta(y,s)\left(\int_0^1 \rho^{\alpha-1}\,dy\right)ds\\ &\leqslant C(T) + C(T)\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds,\end{aligned}$$

where we used Lemmas 3.2–3.5. □

Lemma 3.9. *One has the following inequality*

$$\int_0^t \max_{y\in[0,1]} \theta^{2(1+r)}(y,s)\,ds \leqslant C(T) + C\int_0^t \left(\int_0^1 \frac{\rho(1+\theta^q)\theta_y^2}{\theta^2}\,dy\right) \times \left(\int_0^1 v\,dy\right)ds \tag{3.22}$$

for $0<\alpha\leqslant\frac{1}{2}$ *and* $0<t\leqslant T$.

Proof. By the embedding theorem and Hölder's inequality, we arrive at

$$\theta^{1+r} \leqslant \int_0^1 \theta^{1+r}\,dy + (1+r)\int_0^1 \theta^r|\theta_y|\,dy \leqslant C + C\left(\int_0^1 \frac{\rho\theta^{2+2r}\theta_y^2}{\theta^2}\,dy\right)^{\frac{1}{2}}\left(\int_0^1 v\,dy\right)^{\frac{1}{2}}$$
$$\leqslant C + C\left(\int_0^1 \frac{\rho(1+\theta^q)\theta_y^2}{\theta^2}\,dy\right)^{\frac{1}{2}}\left(\int_0^1 v\,dy\right)^{\frac{1}{2}} \quad (q \geqslant 2+2r). \tag{3.23}$$

Taking square on both sides of (3.23) and integrating it over $[0,t]$, we have

$$\int_0^t \max_{y\in[0,1]} \theta^{2(1+r)}(y,s)\,ds \leqslant C(T) + C\int_0^t \left(\int_0^1 \frac{\rho(1+\theta^q)\theta_y^2}{\theta^2}\,dy\right) \times \left(\int_0^1 v\,dy\right) ds,$$

which completes the proof of the lemma. □

Lemma 3.10. *One has the following inequality*

$$\int_0^1 v\,dy \leqslant C(T), \quad \forall t \in (0,T], \tag{3.24}$$

where $0 < \alpha \leqslant \frac{1}{2}$.

Proof. Integrating (2.6a) over $[0,1]\times[0,t]$ leads to

$$\int_0^1 v\,dy - \int_0^1 v_0(y)\,dy = \int_0^t \big[u(1,s) - u(0,s)\big]\,ds = \int_0^t \big[\big(v^\alpha\rho^\alpha u\big)(1,s) - \big(v^\alpha\rho^\alpha u\big)(0,s)\big]\,ds.$$

From Lemmas 3.6 and 3.8, we deduce

$$\int_0^1 v\,dy \leqslant \int_0^1 v_0(y)\,dy + \int_0^t \big(v^\alpha(0,s) + v^\alpha(1,s)\big)\big\|\rho^\alpha u\big\|_{C[0,1]}\,ds \leqslant C + \left(C(T) + C\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds\right)^2$$
$$\leqslant C(T) + C\left(\int_0^t \max_{y\in[0,1]} \theta^{1+r}(y,s)\,ds\right)^2 \leqslant C(T) + C\int_0^t \max_{y\in[0,1]} \theta^{2(1+r)}(y,s)\,ds. \tag{3.25}$$

Substituting (3.22) into (3.25) and noticing the initial conditions, we have

$$\int_0^1 v\,dy \leqslant C(T) + C\int_0^t \max_{y\in[0,1]} \theta^{2(1+r)}(y,s)\,ds \leqslant C(T) + C\int_0^t \left(\int_0^1 \frac{\rho(1+\theta^q)\theta_y^2}{\theta^2}\,dy\right) \times \left(\int_0^1 v\,dy\right) ds, \tag{3.26}$$

which implies the proof by the Gronwall's inequality and Lemma 3.4. □

Based on Lemma 3.10, we can derive from Lemmas 3.6–3.9 the following estimates.

Lemma 3.11. *One has the following inequalities*

$$\int_0^t \max_{y\in[0,1]} \theta^{2(1+r)}(y,s)\,ds \leqslant C(T), \tag{3.27}$$

$$\int_0^1 \big[\big(\rho^\alpha\big)_y\big]^2\,dy \leqslant C(T), \tag{3.28}$$

and

$$v(d,t) \leqslant C(T), \quad d = 0,1, \tag{3.29}$$

for $0 < \alpha \leqslant \frac{1}{2}$ *and* $0 < t \leqslant T$.

Proof. To show (3.27), collecting (3.22) and (3.24), we have

$$\int_0^t \max_{y\in[0,1]} \theta^{2(1+r)}(y,s)\,ds \leqslant C(T) + C\int_0^t \left(\int_0^1 \frac{(1+\theta^q)\theta_y^2}{v\theta^2}\,dy\right)\times\left(\int_0^1 v\,dy\right)ds$$
$$\leqslant C(T) + C(T)\int_0^t\int_0^1 \frac{\rho(1+\theta^q)\theta_y^2}{\theta^2}\,dy\,ds$$
$$\leqslant C(T),$$

where we used Lemma 3.4.

The proof of (3.28) and (3.29) follows from (3.12), (3.15) and (3.27). □

Lemma 3.12. *Under the conditions of Theorem* 2.1, *it holds for* $0<\alpha\leqslant\frac{1}{2}$,

$$v(y,t)\leqslant C(T) \iff \rho(y,t)\geqslant 1/C(T), \quad \forall (y,t)\in[0,1]\times(0,T]. \tag{3.30}$$

Proof. Let $V(t)=\max_{(y,s)\in[0,1]\times[0,t]} v(y,s)$. By the Sobolev's embedding theorem $W^{1,1}([0,1])\hookrightarrow L^\infty([0,1])$, we obtain for any $0<\delta<1$,

$$v^\delta \leqslant \int_0^1 v^\delta\,dy + \delta\int_0^1 v^{\delta-1}|v_y|\,dy.$$

From Hölder's inequality, Lemma 3.10 and (3.28), we have

$$v^\delta \leqslant \int_0^1 v^\delta\,dy + \delta\int_0^1 v^{\delta-1}|v_y|\,dy \leqslant \left(\int_0^1 v\,dy\right)^\delta + C(T)\delta\left(\int_0^1 \left[(\rho^\alpha)_y\right]^2 dy\right)^{\frac{1}{2}}\left(\int_0^1 v^{2(\delta+\alpha)}\,dy\right)^{\frac{1}{2}}$$
$$\leqslant C(T) + C(T)\delta V^\delta\left(\int_0^1 v^{2\alpha}\,dy\right)^{\frac{1}{2}} \leqslant C(T)+C(T)\delta V^\delta \quad \text{since } 0<2\alpha\leqslant 1.$$

Choosing a sufficiently small δ such that $C(T)\delta<1$, we get $v(y,t)\leqslant C(T)$, which implies the proof of the lemma together with (3.29). □

With the upper and the lower bound of density in hand, we can establish the H^1 norm of the density $\rho(y,t)$, velocity $u(y,t)$ and the reactant mass fraction $z(y,t)$ since the initial data are in H^1 space. On the other hand, by the previous estimates, we can also obtain the lower and upper bound of the temperature. These detailed proofs can be found in [23].

This completes the proof of Theorem 2.1.

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